

Slowly rotating black holes in the Hořava-Lifshitz gravity

Hyung Won Lee, Yong-Wan Kim, Yun Soo Myung

Institute of Basic Science and School of Computer Aided Science,
Inje University, Gimhae 621-749, Korea

Abstract

We investigate slowly rotating black holes in the Hořava-Lifshitz (HL) gravity. For $\Lambda_W = 0$ and $\lambda = 1$, we find a slowly rotating black hole of the Kehagias-Sfetsos solution in asymptotically flat spacetimes. We discuss their thermodynamic properties by computing mass, temperature, angular momentum, and angular velocity on the horizon.

1 Introduction

Hořava has proposed a renormalizable theory of gravity at a Lifshitz point [1, 2], which may be regarded as a UV complete candidate for general relativity. At short distances the theory of Hořava-Lifshitz (HL) gravity describes interacting nonrelativistic gravitons and is supposed to be power counting renormalizable in (1+3) dimensions. Recently, its black hole solutions has been intensively investigated in [4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

Considering spherically symmetric spacetimes, Lü-Mei-Pope (LMP) have obtained the black hole solution with dynamical parameter λ [4] and topological black holes were found in [5]. Its thermodynamics were studied in [11], but there remain unclear issues in defining the ADM mass and entropy because their asymptotes are Lifshitz [8, 9]. On the other hand, Kehagias and Sfetsos (KS) have found the $\lambda = 1$ black hole solution in asymptotically flat spacetimes using the deformed HL gravity [10]. Its thermodynamics was discussed in Ref.[13, 23]. On later, Park has obtained a $\lambda = 1$ black hole solution with two parameter ω and Λ_W [17] and the authors in [24] have found that the BTZ black strings are solutions to the HL gravity. The most general spherically symmetric solution with zero shift vector was found in the non-projectable Hořava-Lifshitz class of theories with general coupling constants for the quadratic terms [25].

It is very curious to find a rotating black hole solution in the HL gravity. However, it seems to be a formidable task to find a fully rotating solution because equations of motion to be solved are very complicated. In this work, we wish to find a slowly rotating black hole solutions based on the KS solutions by introducing a non-zero shift vector. Here “slowly rotating” black hole means that we consider up to linear order of rotating parameter $a = J/M (a \ll 1)$ in the metric functions, equations of motion, and thermodynamic quantities [26, 27, 28, 29, 30, 31]. We mention that the slowly rotating Kerr black hole is recovered from the slowly rotating black hole solutions in the HL gravity, in the IR limit of $\omega \rightarrow \infty (\kappa^2 \rightarrow 0)$.

2 HL gravity

In this section, we review briefly the HL gravity including the soft violation term. In the ADM formalism, the four dimensional metric of general relativity is parameterized as [32]

$$ds_4^2 = -N^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt), \quad (1)$$

where the lapse, shift and 3-metric N , N^i , and g_{ij} are all functions of t and x^i . In the simplest version of the theory which respects the projectability condition [1, 2], the lapse function N is viewed as a gauge field for time re-parameterizations, and it is effectively restricted to depend only on t . A closer parallel with general relativity could be achieved if this projectability restriction is relaxed. Thus one may take a broader view of the Hořava proposal without this condition as an interesting class of theories. Then, the relative coefficients of the terms in the ADM decomposition of the Einstein-Hilbert action are modified, and additional terms involving higher spatial derivatives are included too. The higher derivative terms may improve the renormalizability of the theory without the problems of ghosts that would arise if higher temporal derivatives were present.

The ADM decomposition of the Einstein-Hilbert action is given by

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{g} N (K_{ij} K^{ij} - K^2 + R - 2\Lambda), \quad (2)$$

where G is Newton's constant and extrinsic curvature K_{ij} is defined by

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i). \quad (3)$$

Here, a dot denotes a derivative with respect to t .

The modified action of the theory including $\mu^4 R$ can be written as

$$S = \int dt d^3\mathbf{x} (\mathcal{L}_0 + \mathcal{L}_1), \quad (4)$$

$$\mathcal{L}_0 = \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda_W R - 3\Lambda_W^2)}{8(1-3\lambda)} + \frac{\kappa^2 \mu^2 \omega R}{8(3\lambda-1)} \right\},$$

$$\mathcal{L}_1 = \sqrt{g} N \left\{ \frac{\kappa^2 \mu^2 (1-4\lambda)}{32(1-3\lambda)} R^2 - \frac{\kappa^2}{2W^4} \left(C_{ij} - \frac{\mu W^2}{2} R_{ij} \right) \left(C^{ij} - \frac{\mu W^2}{2} R^{ij} \right) \right\},$$

where λ, κ, μ , and W are constant parameters to denote the HL gravity and Λ_W is a negative cosmological constant. Here $\omega = 8(3\lambda-1)\mu^2/\kappa^2$ is a parameter to represent a soft violation term of the detailed balance condition, $\mu^4 R$ and C_{ij} is the Cotton tensor defined by

$$C^{ij} = \epsilon^{ik\ell} \nabla_k \left(R^j_\ell - \frac{1}{4} R \delta^j_\ell \right). \quad (5)$$

Two cases of $\Lambda_W = 0$ with $\lambda = 1$ and $\omega = 0$ are included: the former case provides the KS solution, while the latter shows the LMP solution. For the case of $\omega = 0$, comparing \mathcal{L}_0 to that of general relativity plus negative

cosmological constant in the ADM formalism, the speed of light, Newton's constant, and the cosmological constant emerge as [1]

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1 - 3\lambda}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda = \frac{3}{2}\Lambda_W \quad (6)$$

which shows asymptotically AdS spacetimes. On the other hand, for $\Lambda_W = 0$ with $\lambda = 1$, we have asymptotically flat spacetimes as [10]

$$c^2 = \frac{\kappa^2 \mu^4}{2}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda = 0. \quad (7)$$

Since we wish to find a non-spherically symmetric solution of black hole, it needs to derive full equations of motion from the action (4) [4]. The equations of motion were also obtained in [3]. In deriving full equations, we have relaxed both the projectability restriction and detailed balance condition since the lapse function N depends on the spatial coordinates x^i as well as a soft violation term of $\mu^4 R$ is included.

3 Slowly rotating black hole

In this section, we find a slowly rotating black hole in asymptotically flat spacetimes. Before we proceed, we would like to mention a static solution, the KS solution with $\Lambda_W = 0$ and $\lambda = 1$. In this case, equation of motion for the lapse function N can be read as

$$\frac{2}{\kappa^2}(K_{ij}K^{ij} - K^2) + \mu^4 R + \frac{3\kappa^2 \mu^2}{64}R^2 - \frac{\kappa^2}{2W^4}Z_{ij}Z^{ij} = 0 \quad (8)$$

with

$$Z_{ij} = C_{ij} - \frac{\mu W^2}{2}R_{ij}. \quad (9)$$

The variation with respect to δN^i implies

$$\nabla_k(K^{k\ell} - K g^{k\ell}) = 0. \quad (10)$$

The equations of motion from variation of δg^{ij} takes the form

$$\frac{2}{\kappa^2}E_{ij}^{(1)} - \frac{2}{\kappa^2}E_{ij}^{(2)} + \mu^4 E_{ij}^{(3)} + \frac{3\kappa^2 \mu^2}{64}E_{ij}^{(4)} - \frac{\mu \kappa^2}{4W^2}E_{ij}^{(5)} - \frac{\kappa^2}{2W^4}E_{ij}^{(6)} = 0, \quad (11)$$

where all $E_{ij}^{(r)}$ for $r = 1, \dots, 6$ are the same as in Ref.[4] except the case

$$E_{ij}^{(3)} = N\left(R_{ij} - \frac{1}{2}Rg_{ij}\right) - (\nabla_i \nabla_j - g_{ij} \nabla_k \nabla^k)N. \quad (12)$$

A spherically symmetric solution to these equations was obtained by considering the line element

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (13)$$

In this case, we have $K_{ij} = 0$ and $C_{ij} = 0$. The KS solution is given by

$$f_{\text{KS}} = N_{\text{KS}}^2 = 1 + \omega r^2 \left(1 - \sqrt{1 + \frac{4M}{\omega r^3}} \right) \quad (14)$$

with $\omega = 16\mu^2/\kappa^2$ and the mass parameter M . In the limit of $\omega \rightarrow \infty$ (equivalently, the IR limit of $\kappa^2 \rightarrow 0$), it reduces to the Schwarzschild form of

$$f_{\text{Sch}}(r) = 1 - \frac{2M_{\text{Sch}}}{r}. \quad (15)$$

Now let us introduce an axisymmetric metric ansatz with one component shift vector (or shift function) $N^\phi(r)$ as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta \left[d\phi - aN^\phi(r)dt \right]^2 \quad (16)$$

with the rotation parameter $a = J/M$. Note that even the above metric contains up to the second order of a , we will keep equations of motion up to the linear order of a in order to obtain a slowly rotating black hole solution. This is equivalent to considering the slowly rotating metric initially

$$ds_{\text{slow R}}^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta \left[d\phi^2 - 2aN^\phi(r)dt d\phi \right]. \quad (17)$$

For an axisymmetric metric ansatz, we cannot use the reduced Lagrangian approach which was developed in [4] to find f and N because (17) contains a non-singlet function N^ϕ under the $\text{SO}(3)$ action on the S^2 . Hence, we have to solve Eqs.(8), (10), and (11), simultaneously.

Using Eq.(16), non-zero components of extrinsic curvature are found to be

$$K_{ij} = \begin{pmatrix} 0 & 0 & -\frac{1}{2} \frac{r^2 a \sin^2 \theta}{\sqrt{f(r)}} \frac{dN^\phi(r)}{dr} \\ 0 & 0 & 0 \\ -\frac{1}{2} \frac{r^2 a \sin^2 \theta}{\sqrt{f(r)}} \frac{dN^\phi(r)}{dr} & 0 & 0 \end{pmatrix}, \quad (18)$$

where we observe that $K_{r\phi} = K_{\phi r}$ are linear order of a and its trace is zero ($K = 0$). Also we show that all components of Cotton tensor still vanish

($C_{ij} = 0$) up to linear order of a . This may imply that if one wishes to find a fully rotating black hole, all higher order terms of a must be included. Then, Eq.(10) reduces to

$$\nabla_k K^{k\ell} = \text{diag} \left[0, 0, \frac{a\sqrt{f(r)}}{2r} \left(r \frac{d^2 N^\phi(r)}{dr^2} + 4 \frac{dN^\phi(r)}{dr} \right) \right] = 0, \quad (19)$$

which has a solution with two arbitrary constants C_1 and C_2

$$N^\phi(r) = C_1 + \frac{C_2}{r^3}. \quad (20)$$

For later convenience, we choose the shift function to be

$$N^\phi(r) = \frac{2M}{r^3} \quad (21)$$

with $C_1 = 0$ and $C_2 = 2M$. In this case, one has non-zero $g_{t\phi}$ and $g_{\phi t}$ components

$$g_{t\phi} = g_{\phi t} = -ar^2 N^\phi(r) \sin^2 \theta = -\frac{2J}{r} \sin^2 \theta. \quad (22)$$

Plugging these into Eq.(8) leads to

$$\frac{(f-1)^2}{r^2} - \frac{2(f-1)f'}{r} - 2\omega(1-f-rf') = \frac{32a^2 M^2 \sin^2 \theta}{\kappa^4 \mu^4 r^4}. \quad (23)$$

We note that the right-hand side is taken to be zero effectively because it is second order of a . Then, the solution is given by

$$f_{\pm}(r) = 1 + \omega r^2 \left(1 \pm \sqrt{1 + \frac{C_3}{\omega r^3}} \right). \quad (24)$$

A metric function $f_-(r)$ recovers the Schwarzschild black hole solution in the limit of $\omega \rightarrow \infty$. The (r, r) -component of Eq.(11) gives the same equation as in (23). Other two components of (θ, θ) and (ϕ, ϕ) provide second order differential equations for the metric function $f(r)$ and solving these leads to the solution up to linear order a as

$$f_{\pm}(r) = 1 + \omega r^2 \left(1 \pm \sqrt{1 + \frac{2}{\omega r^2} \left(1 - \frac{C_4}{16\mu^2} \right) + \frac{C_5}{\omega r^3}} \right) \quad (25)$$

which becomes the same function as in (24) when choosing $C_4 = 16\mu^2$ and $C_5 = 4M$. This implies that the metric function $f_-(r)$ becomes the KS

solution $f_{\text{KS}}(r)$ in Eq.(14) for slowly rotating black hole solution. Therefore, our slowly rotating black hole solution is given by

$$ds_{\text{slow KS}}^2 = -f_{\text{KS}}(r)dt^2 + \frac{dr^2}{f_{\text{KS}}(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta \left(d\phi^2 - \frac{4J}{r^3} dt d\phi \right). \quad (26)$$

This is our main result.

On the other hand, the Kerr black hole is given by

$$ds_{\text{Kerr}}^2 = -\frac{\rho^2 \Delta_r}{\Sigma^2} dt^2 + \frac{\rho^2}{\Delta_r} dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2 \sin^2 \theta}{\rho^2} (d\phi - \xi dt)^2, \quad (27)$$

where

$$\begin{aligned} \rho^2 &= r^2 + a^2 \cos^2 \theta, \\ \Delta_r &= (r^2 + a^2) - 2Mr, \\ \Sigma^2 &= (r^2 + a^2)^2 - a^2 \sin^2 \theta \Delta_r, \\ \xi &= \frac{2Mar}{\Sigma^2}. \end{aligned} \quad (28)$$

In the slowly rotating limit of $J \ll M(a \ll 1)$, the Kerr solution reduces to

$$ds_{\text{slow Kerr}}^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta \left(d\phi^2 - \frac{4J}{r^3} dt d\phi \right) \quad (29)$$

whose metric is given by

$$g_{\mu\nu}^{\text{slow Kerr}} = g_{\mu\nu}^{\text{Sch}} - \frac{2Ma}{r} \sin^2 \theta \delta_\mu^t \delta_\nu^\phi. \quad (30)$$

This means that the slowly rotating Kerr black hole could be interpreted as arising from the breaking of spherical to axial symmetry. It is easily checked that in the limit of $\omega \rightarrow \infty$, the slowly rotating solution (26) leads to the slowly rotating Kerr solution (29).

Since we have still the KS metric function $f_{\text{KS}}(r)$ for slowly rotating black hole, we use the horizon mass M_h and temperature T_H for the KS black hole as [33, 34]

$$M_h(r_+, \omega) = \frac{r_+}{2} - \frac{3 \tan^{-1}(\sqrt{\omega} r_+)}{4\sqrt{\omega}}, \quad T_H = \frac{2\omega r_+^2 - 1}{8\pi r_+ (\omega r_+^2 + 1)}, \quad (31)$$

where r_+ is the outer horizon as a root of $f_{\text{KS}}(r_+) = 0$. Importantly, we have angular velocity defined by

$$\Omega = -\frac{g_{t\phi}}{g_{\phi\phi}} = aN^\phi = \frac{2J}{r^3}, \quad (32)$$

which is the same form as that of the slowly rotating Kerr black hole. The angular velocity on the horizon is given by

$$\Omega_h = \frac{2M_h a}{r_+^3} = a \left[\frac{1}{r_+^2} - \frac{3 \tan^{-1}(\sqrt{\omega} r_+)}{2\sqrt{\omega} r_+^3} \right] \quad (33)$$

which reduces, in the limit of $\omega \rightarrow \infty$, to the angular velocity of the slowly rotating Kerr black hole on the horizon

$$\Omega_h^{\text{Kerr}} = \frac{a}{r_+^2} = \frac{2J}{r_+^3} \quad (34)$$

with $M_{\text{Sch}} = r_+/2$. Finally, the angular momentum of slowly rotating black hole is given by

$$J = aM_h = a \left[\frac{r_+}{2} - \frac{3 \tan^{-1}(\sqrt{\omega} r_+)}{4\sqrt{\omega}} \right]. \quad (35)$$

In the limit of $\omega \rightarrow \infty$, it leads to the angular momentum of slowly rotating Kerr black hole

$$J \rightarrow J^{\text{slow Kerr}} = \frac{a r_+}{2}. \quad (36)$$

If one uses the mass parameter defined in (14) [13] instead of the horizon mass M_h , it takes the form

$$M(r_+, \omega) = \frac{r_+}{2} + \frac{1}{4\omega r_+}. \quad (37)$$

Here we observe an inequality

$$M_h < M_{\text{Sch}} < M, \quad (38)$$

but they become the same form in the limit of $\omega \rightarrow \infty$.

4 Discussions

It seems to be a formidable task to find a fully rotating black hole in the HL gravity because full equations to be solved are very complicated. In this work, we have found the slowly rotating black hole solution based on the KS solution in the HL gravity.

First of all, we explain why the slowly rotating solution is naturally obtained for the HL gravity by examining the order of rotation parameter a

in equations of motion. The axisymmetric metric ansatz (16) was implemented by one component shift vector, N^ϕ . Hence, extrinsic curvature K_{ij} has off-diagonal components as shown in (18). This implies that equation (8) obtained from variation of lapse function N remains unchanged when adding a rotating parameter term to the spherically symmetric case. This is confirmed by showing that $K = 0$ and $K_{ij} = 0$ for spherically symmetric case, while for slowly rotating case, $K = 0$ and $K_{ij}K^{ij} = \mathcal{O}(a^2)$. Effectively, Eq.(8) is the same equation for both two cases. A shift vector N^ϕ could be determined by Eq.(10). We emphasize that other equations remain unchanged at linear order of a . It is clear that $E_{ij}^{(1)} = 0$ for spherically symmetric case and $E_{ij}^{(1)} = \mathcal{O}(a^2)$ for slowly rotating case, which is effectively taken to be zero. $E_{ij}^{(2)} = 0$ for both two cases. All other $E_{ij}^{(r)}$ for $r = 3, \dots, 6$ remain unchanged, since they contain terms derived from g_{ij} which do not carry rotation parameter a , and $C_{ij} = 0$ by rotation symmetry in 3D Euclidean space.

In summary, all equations of motion remain unchanged by introducing a slowly rotating parameter a except Eq. (10), which was zero for spherically symmetric case. The slowly rotating black hole could be interpreted as arising from the breaking of spherical to axial symmetry.

Finally, we propose a general axisymmetric metric ansatz for a slowly rotating black hole

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta \left[d\phi^2 - 2aN^\phi(r) dt d\phi \right] \quad (39)$$

with the shift vector

$$N^\phi(r) = C_1 + C_2 \int^r \frac{N(r')}{r'^4 \sqrt{f(r')}} dr'. \quad (40)$$

Hence, for any spherically symmetric solution, we always construct a slowly rotating solution by introducing a non-zero shift function determined by Eq. (40). Considering (39), any spherically symmetric solutions may be also candidates for constructing slowly rotating black hole solutions in the HL gravity. However, it seems difficult to seek the slowly rotating black holes of the LMP black hole solutions [4] because these solutions include the $\lambda = 1$ AdS black hole with double horizons, $\lambda = 1/2$ and $9/25$ Lifshitz black holes with single horizon, and $\lambda \geq 3$ Reissner-Nordström-type black holes with double horizons [9].

As another example, we would like to mention that the KS black hole could be also the solution to modified $F(R)$ HL gravity [35, 36, 37]. Especially, in order to find the KS black hole solution, the non-projectable

modified $F(R)$ HL action in [37]

$$S_F = \frac{1}{\kappa^2} \int dt d^3x \sqrt{g} N F({}^4\tilde{R}), \quad (41)$$

$${}^4\tilde{R} = K_{ij}K^{ij} - \lambda K^2 + 2\tilde{\mu}\nabla_\mu(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu) - \mathcal{L}_R(g_{ij}) \quad (42)$$

could be transformed into the KS action [33]

$$\mu^4 \int dt d^3x \sqrt{g} N \tilde{\mathcal{L}}_V = \mu^4 \int dt d^3x \sqrt{g} N \left[R - \frac{2}{\omega} \left(R_{ij}R_{ij} - \frac{3}{8}R^2 \right) \right], \quad (43)$$

by adjusting the parameters $\alpha_i (i = 1, \dots, 8)$ in $\mathcal{L}_R(g_{ij})$ of (2.55) and taking $F({}^4\tilde{R}) = {}^4\tilde{R}$, $K_{ij} = \tilde{\mu} = 0$, and $1/\kappa^2 = \mu^4$. In this case, we propose that our solution (26) is also the slowly rotating solution to $F(R)$ HL gravity. It is interesting to find a spherically symmetric solution for $F({}^4\tilde{R}) \neq {}^4\tilde{R}$ case and its slowly rotating solution.

Acknowledgement

This work was supported by Basic Science Research Program through the National Research Foundation (NRF) of Korea funded by the Ministry of Education, Science and Technology (2009-0086861).

References

- [1] P. Horava, Phys. Rev. D **79** (2009) 084008 [arXiv:0901.3775 [hep-th]].
- [2] P. Horava, JHEP **0903** (2009) 020 [arXiv:0812.4287 [hep-th]];
M. Visser, Phys. Rev. D **80** (2009) 025011 [arXiv:0902.0590 [hep-th]];
P. Horava, Phys. Rev. Lett. **102** (2009) 161301 [arXiv:0902.3657 [hep-th]].
- [3] E. Kiritsis and G. Kofinas, Nucl. Phys. B **821** (2009) 467 [arXiv:0904.1334 [hep-th]].
- [4] H. Lu, J. Mei and C. N. Pope, Phys. Rev. Lett. **103** (2009) 091301 [arXiv:0904.1595 [hep-th]].
- [5] R. G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D **80** (2009) 024003 [arXiv:0904.3670 [hep-th]].

- [6] R. G. Cai, Y. Liu and Y. W. Sun, JHEP **0906** (2009) 010 [arXiv:0904.4104 [hep-th]].
- [7] E. O. Colgain and H. Yavartanoo, JHEP **0908** (2009) 021 [arXiv:0904.4357 [hep-th]].
- [8] Y. S. Myung and Y. W. Kim, Eur. Phys. J. C **68** (2010) 265 [arXiv:0905.0179 [hep-th]].
- [9] Y. S. Myung, Phys. Lett. B **690** (2010) 534 [arXiv:1002.4448 [hep-th]].
- [10] A. Kehagias and K. Sfetsos, Phys. Lett. B **678** (2009) 123 [arXiv:0905.0477 [hep-th]].
- [11] R. G. Cai, L. M. Cao and N. Ohta, Phys. Lett. B **679** (2009) 504 [arXiv:0905.0751 [hep-th]].
- [12] A. Ghodsi, arXiv:0905.0836 [hep-th].
- [13] Y. S. Myung, Phys. Lett. B **678** (2009) 127 [arXiv:0905.0957 [hep-th]].
- [14] S. Chen and J. Jing, Phys. Lett. B **687** (2010) 124 [arXiv:0905.1409 [gr-qc]].
- [15] S. b. Chen and J. l. Jing, Phys. Rev. D **80** (2009) 024036 [arXiv:0905.2055 [gr-qc]].
- [16] J. Chen and Y. Wang, Int. J. Mod. Phys. A **25** (2010) 1439 [arXiv:0905.2786 [gr-qc]].
- [17] M. i. Park, JHEP **0909** (2009) 123 [arXiv:0905.4480 [hep-th]].
- [18] M. Botta-Cantcheff, N. Grandi and M. Sturla, arXiv:0906.0582 [hep-th].
- [19] A. Ghodsi and E. Hatefi, Phys. Rev. D **81** (2010) 044016 [arXiv:0906.1237 [hep-th]].
- [20] A. Castillo and A. Larranaga, arXiv:0906.4380 [gr-qc].
- [21] J. J. F. Peng and S. Q. F. Wu, Eur. Phys. J. C **66** (2010) 325 [arXiv:0906.5121 [hep-th]].
- [22] H. W. Lee, Y. W. Kim and Y. S. Myung, Eur. Phys. J. C **68** (2010) 255 [arXiv:0907.3568 [hep-th]].

- [23] Y. S. Myung, Phys. Lett. B **684** (2010) 158 [arXiv:0908.4132 [hep-th]].
- [24] I. Cho and G. Kang, JHEP **1007** (2010) 034 [arXiv:0909.3065 [hep-th]].
- [25] E. Kiritsis and G. Kofinas, JHEP **1001** (2010) 122 [arXiv:0910.5487 [hep-th]].
- [26] J. H. Horne and G. T. Horowitz, Phys. Rev. D **46** (1992) 1340 [arXiv:hep-th/9203083].
- [27] E. Poisson, Phys. Rev. D **48** (1993) 1860.
- [28] E. A. Martinez, Phys. Rev. D **50** (1994) 4920 [arXiv:gr-qc/9405033].
- [29] T. Ghosh and S. SenGupta, Phys. Rev. D **76** (2007) 087504 [arXiv:0709.2754 [hep-th]].
- [30] H. C. Kim and R. G. Cai, Phys. Rev. D **77** (2008) 024045 [arXiv:0711.0885 [hep-th]].
- [31] Y. S. Myung, Phys. Lett. B **689** (2010) 42 [arXiv:1003.3519 [hep-th]].
- [32] R.L. Arnowitt, S. Deser and C.W. Misner, *The dynamics of general relativity*, “Gravitation: an introduction to current research”, Louis Witten ed. (Wiley 1962), chapter 7, pp 227-265, arXiv:gr-qc/0405109.
- [33] Y. S. Myung, Phys. Lett. B **685** (2010) 318 [arXiv:0912.3305 [hep-th]].
- [34] M. Wang, J. Jing, C. Ding and S. Chen, Phys. Rev. D **81** (2010) 083006 [arXiv:0912.4832 [gr-qc]].
- [35] M. Chaichian, S. Nojiri, S. D. Odintsov, M. Oksanen and A. Tureanu, Class. Quant. Grav. **27** (2010) 185021 [arXiv:1001.4102 [hep-th]].
- [36] S. Carloni, M. Chaichian, S. Nojiri, S. D. Odintsov, M. Oksanen and A. Tureanu, arXiv:1003.3925 [hep-th].
- [37] M. Chaichian, M. Oksanen and A. Tureanu, arXiv:1006.3235 [hep-th].